EXPERIMENTAL MEASUREMENT OF AMPLITUDE CORRECTIONS TO THE RATE OF PROPAGATION OF CAPILLARY WAVES

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In a number of articles devoted to analysis of steady-state capillary-gravity waves on the surface of a liquid (see for example [1-3]), it is shown that the rate of propagation of a nonlinear wave depends on its amplitude. This relationship is expressed by a correction to the rate of propagation of the main harmonic in a dispersing medium and has an order of magnitude $\sim (ka)^2$ (k is the wave vector, and a is the amplitude of the surface wave). For the speed of waves on the surface of an ideal, infinitely deep liquid we can write

$$c = c_0 [1 + \alpha (ka)^2] \tag{1}$$

where c_0 is the speed of a wave where $ka \to 0$, $\alpha = 1/2$ for gravitational waves, and $\alpha = -1/16$ for capillary waves. A method for determining the relationship (1) experimentally for capillary waves is given below.

It is known that in media in which condition (1) is fulfilled, it is possible to achieve such phenomena, which have been thoroughly investigated for electromagnetic waves, as three-dimensional and transient compression (or atomization) of wave packets [4]. However, owing to the presence of a marked damping of capillary waves, the use of these phenomena to verify formula (1) has become more difficult. The nonlinear nature of this damping leads to the fact that when surface waves are excited, a movement of the liquid occurs in the direction of propagation of the wave [5], similar to the acoustic wind in a viscous medium. In practice this effect appears already at values of $a/\lambda \sim 0.01$ and determines the variation in the length of the propagating wave. Moreover at approximately these values of a/λ, the excitation of a sufficiently wide wave beam is accompanied by the occurrence of parametric generation (of the lateral structure of the wave of a subharmonic close to the wave producer), which distorts the picture of the traveling waves.

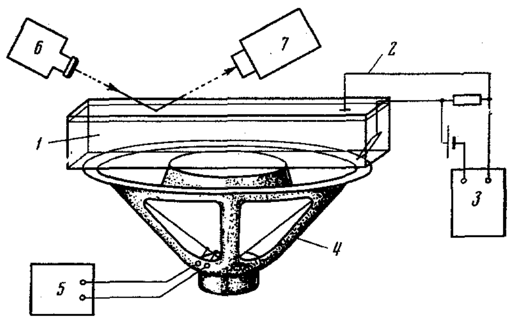

Fig. 1

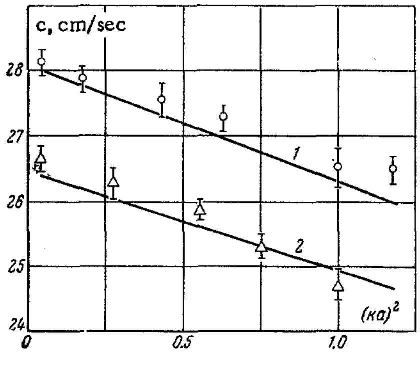

Fig. 2

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The system, the layout of which is given in Fig. 1, enables the above-mentioned difficulties to be avoided. Here 1 is the vessel with the liquid, 2 is the probe, 3 is the frequency meter, 4 is the sound dynamic system, 5 is the sound frequency generator, 6 is the gas laser, and 7 is the reciver. Parametric vibrations with a frequency equal to half the frequency of pumping occur in a narrow rectangular channel filled with liquid and vibrating in a vertical direction. As shown in [8], the vibrations of the subharmonic represent two surface waves traveling to meet one another; these form a standing wave in the region remote from the ends. The chosen length of the channel is sufficiently great (equal to about 50 times the length of the waves of the subharmonic), and hence the threshold value of the amplitude of pumping was determined basically by the magnitude of damping of the surface waves. The amplitude of the surface waves varied within wide limits in proportion to the variation of the amplitude of pumping. The presence of reflecting walls enabled flows of liquid to be fully eliminated.

In accordance with formula (1), when the amplitude of the standing wave increases, the number of half-waves stacked on the length of the resonator varies. The relationship a/λ in the standing wave is measured by an optical method described in [7]. The accuracy of this magnitude determined from the angle of the maximum deflection of the gas laser beam reflected from the surface of the liquid is several percent. The length of the wave in the resonator λ_p is measured by photographing a picture of the standing waves in the channel and by further treatment of the photographs obtained on a microphotometer. In each series, consisting of five measurements, the length of the wave is determined as a result of averaging to a fiftieth of the half waves of the subharmonic, each of which is determined in turn as the distance between adjacent minima. The constancy of the frequency of the vibrations is controlled by using an electronic frequency meter connected with the vibrating surface by means of an electrolytic probe. The measurements are carried out in channels of different width, and it was hence established that the influence of the side walls on the results obtained can be neglected.

The relationships between the speed of the wave and the amplitude for waves on the surface of water (curve 1) and xylene (curve 2) are given in Fig. 2. The downwards sloping relationship $c = c\{(ka)^2\}$, which is close to a straight line is characteristic. The data of Fig. 2 refer to the frequency of the waves of the subharmonic $f = 55$ Hz, but a similar relationship $c = c\{(ka)^2\}$ is also observed for other frequencies in the range 30-200 Hz. The difference between the experimental data and the theoretical data expressed according to the formula

$$c = \sqrt{\sigma k / \rho}\,[1 - {}^{1}/_{16}\,(ka)^2]$$

is associated in the first place with the fact that tabular values of the surface tension coefficient σ are used. If σ of real liquids is determined by measuring the length of the wave at small a/λ, that is, by combining the initial experimental points and the points of the straight lines in Fig. 2, then the agreement of the theoretical and experimental results improves. The method given enables the correction to the speed to be measured right up to values of $a/\lambda \sim 0.1$. Further increase of the amplitudes of capillary waves is limited by distortion of the shape of the standing wave and the appearance of instability associated with the appearance of parametric generation of a lateral wave in the channel.

In conclusion it must be stressed that the negative sign in formula (1) must lead to automatic focusing of the final capillary wave beam (in this sense the surface of the water is equivalent to the optical medium in which the amplitude correction [4] to the dielectric permeability $\Delta\varepsilon > 0$). However, a breaking up of the initial beam, which is characteristic for the defocusing medium with $\Delta\varepsilon < 0$, is observed experimentally. This effect, which is not taken into account in [1-3], as we have pointed out above, is associated with the occurrence of an intense flow of liquid in the direction of propagation of the surface wave.

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